

9. (a) Solve the following simultaneous equation : 15

$$\frac{dx}{dt} + 2x - 3y = 5t ; \frac{dy}{dt} - 3x + 2y = 2e^{2t}$$

(b) An e.m.f. $E \sin pt$ is applied at $t = 0$ to a circuit containing a capacitance C and inductance L . The current i satisfies the equation

$L \frac{di}{dt} + \frac{1}{C} \int i dt = E \sin pt$. If $p^2 = \frac{1}{LC}$ and initially the current i and the charge q are zero, show that the current at time t is $(\frac{Et}{2L}) \sin pt$, where $i = \frac{dq}{dt}$.

Roll No.

3034

B. Tech. 3rd Semester (CSE) Examination – March, 2021

MATHEMATICS - III (Multivariable Calculus and
Differential Equations)

Paper : BSC-MATH-203-G

Time : Three Hours]

[Maximum Marks : 75

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt five questions in all, selecting one question from each Unit. Question No. 1 is compulsory. All questions carry equal marks.

1. (a) Evaluate $Lt \frac{xy}{x^2 + y^2}$. 15
 $\begin{matrix} x \rightarrow 0 \\ y \rightarrow 0 \end{matrix}$

(b) If $u = x^2 + y^2 + z^2$ and $x = e^{2t}$, $y = e^{2t} \cos 3t$ and $z = e^{2t} \sin 3t$. Find $\frac{du}{dt}$.

(c) Evaluate $\iint r \sin \theta dr d\theta$ over the cardioid $r = a(1 - \cos \theta)$ above the initial line.

(d) Solve $x \log x \frac{dy}{dx} + y = \log x^2$.

(e) Solve $\frac{d^4 y}{dx^4} + a^4 y = 0$

(f) Solve $\frac{d^2 y}{dx^2} + y = \tan x$ by the method of variation of parameter.

UNIT - I

2. (a) If $x^x y^y z^z = C$, show that at $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$ 15

(b) If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$, prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \cos 3u \sin u$$

3. (a) Find the minimum value of $x^2 + y^2 + z^2$, given that $xyz = a^3$. 15

(b) If $u = a^3 x^{-2} + b^3 y^{-2} + c^3 z^{-2}$, where $x^{-1} + y^{-1} + z^{-1} = 1$, find the stationary value of u .

UNIT - II

4. (a) Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} x^3 y dx dy$ by changing the order of integration. 15

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(b) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ by using the transformation $x = r \cos \theta$ and $y = r \sin \theta$.

5. (a) Find by double integration, the area lying inside the circle $r = a \sin \theta$ and outside the cardioid $r = a(1 - \cos \theta)$. 15

(b) Evaluate $\int_0^a \int_0^{a-x} \int_0^{a-x-y} e^{x+y+z} dz dy dx$

UNIT - III

6. (a) Solve $\frac{dy}{dx} - \tan y = (1+x)e^x \sec y$. 15

(b) Solve $(x^2 y^2 + xy + 1)y dx + (x^2 y^2 - xy + 1)x dy = 0$

7. (a) A steam pipe 20 cm in diameter contains steam at 150° C and is covered with asbestos 5 cm thick. The outside temp is kept at 60°C. By how much the thickness of the covering be increased in order that the rate of heat loss should be decreased by 25%? 15

(b) Find the orthogonal trajectories of the family of confocal and coaxial parabolas $r = 2a/(1 + \cos \theta)$.

UNIT - IV

8. (a) Solve $(D^3 - D)y = 2x + 1 + 4 \cos x + 2e^x$ 15

(b) Solve $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^2 + 2 \log x$

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P. T. O.